Substituting from the original equation and (3) into (2), we have

$$x\frac{dy}{dx} + \tfrac{1}{2}y = \frac{y}{x}. \tag{4}$$

From this we have

$$\log y = -1/x - \tfrac{1}{2}\log x + \log C,$$

or

$$y = Ce^{-(1/x)}x^{-\frac{1}{2}}.$$

II. Solution by the Proposer.

According to Professor Kelland (*Trans. Royal Society of Edinburgh*, Vols. XIV and XVI) the general differential operator may be defined as follows:

$$\frac{d^\mu x^n}{dx^\mu} = (-1)^\mu \frac{\Gamma(-n+\mu)}{\Gamma(-n)} x^{n-\mu},$$

for all values of n and μ. We have $\Gamma(n+1) = n\Gamma(n)$.

Let us assume $y = A_0 + A_1x^{-\frac{1}{2}} + A_2x^{-1} + A_3x^{-\frac{3}{2}} + \cdots$.

Then

$$\frac{d^{\frac{1}{2}}y}{dx^{\frac{1}{2}}} = (-1)^{\frac{1}{2}}\left\{\frac{\Gamma(1)}{\Gamma(\frac{1}{2})}A_1x^{-1} + \frac{\Gamma(\frac{3}{2})}{\Gamma(1)}A_2x^{-\frac{3}{2}} + \frac{\Gamma(2)}{\Gamma(\frac{3}{2})}A_3x^{-2} + \cdots\right\}$$

and

$$\frac{y}{x} = A_0x^{-1} + A_1x^{-\frac{3}{2}} + A_2x^{-2} + A_3x^{-\frac{5}{2}} + \cdots.$$

Hence,

$$i\frac{\Gamma(1)}{\Gamma(\frac{1}{2})}A_1 = A_0 \text{ and } A_1 = \frac{\sqrt{\pi}}{i}A_0 = -i\sqrt{\pi}A_0.$$

For, since $\Gamma(p)\Gamma(1-p) = \pi/\sin p\pi$ when p is a fraction less than one, then $\Gamma(\frac{1}{2}) = \sqrt{\pi}$. Also, $\Gamma(1) = 1$.

Furthermore,

$$i\frac{\Gamma(\frac{3}{2})}{\Gamma(1)}A_2 = A_1 = -i\sqrt{\pi}A_0, \text{ or } A_2 = -2A_0,$$

since $\Gamma(\frac{3}{2}) = \frac{1}{2}\Gamma(\frac{1}{2}) = \frac{1}{2}\sqrt{\pi}$; also

$$i\frac{\Gamma(2)}{\Gamma(\frac{3}{2})}A_3 = A_2 = -2A_0, \text{ or } A_3 = -\frac{\sqrt{\pi}}{i}A_0 = i\sqrt{\pi}A_0.$$

Finally, $y = A_0(1 - i\sqrt{\pi}x^{-\frac{1}{2}} - 2x^{-1} + i\sqrt{\pi}x^{-\frac{3}{2}} + \cdots)$.

Note.—This problem, which is very similar to one solved by Professor Kelland, was submitted because it was thought that it might prove of interest to the readers of the Monthly. The proposer would be glad to see some discussion of the subject of general differentiation.

The above solution may be considered as a reply to the following question:

360 (Calculus.) Proposed by ELMER SCHUYLER, Brooklyn, New York.

What interpretation must be given to

$$\frac{d^{\frac{1}{2}}y}{dx^{\frac{1}{2}}} \text{ so that } \frac{d^{\frac{1}{2}}}{dx^{\frac{1}{2}}}\left(\frac{d^{\frac{1}{2}}y}{dx^{\frac{1}{2}}}\right) = \frac{dy}{dx}?$$

435 (Calculus). Proposed by B. F. FINKEL, Drury College.

Show that

$$\int_0^\infty e^{-x^2-a^2/x^2}dx = \frac{\sqrt{\pi}}{2e^{2a}}$$

by a transformation, rather than by the usual method of differentiating under the sign of integration, as, for example, in Byerly's *Integral Calculus*, page 106–107.